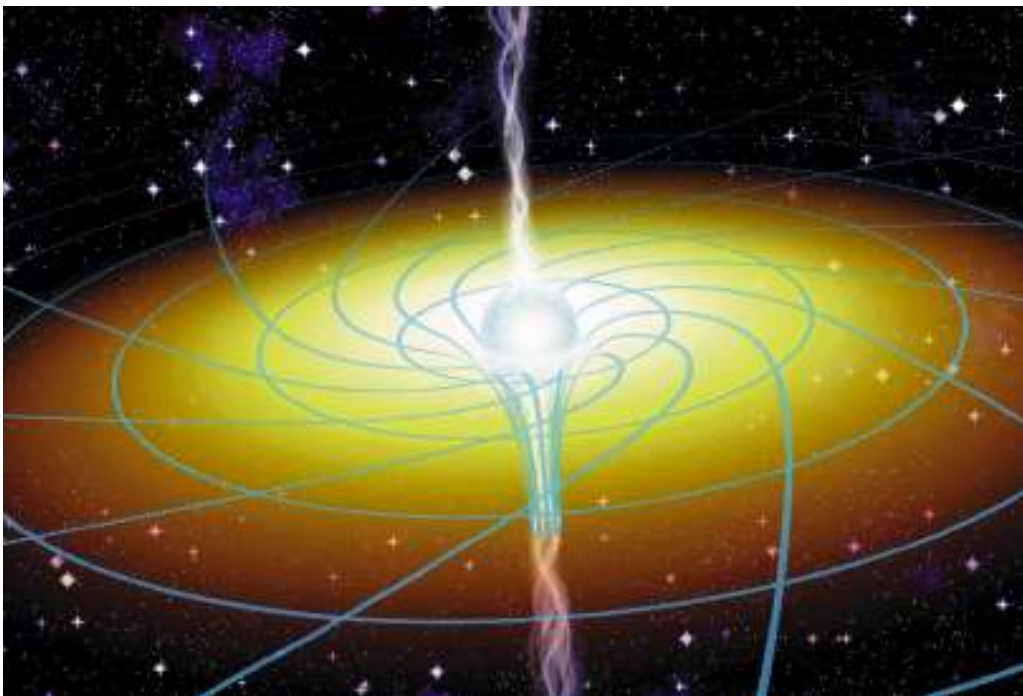


BLACK HOLES

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Honors Physics



Characteristics and Properties of Schwarzschild Black Holes

Many people have heard of the strange celestial objects called black holes, but only a small percentage of them really know what they are. Some people have a vague image in their mind of a huge black “hole” in space sucking tons of things into it, swirling and destroying everything. While this image is somewhat accurate, it gives nothing as to the structure or reality of these objects. There are two kinds of black holes theorized. One is the Schwarzschild, which is a non-rotating, non-charged, symmetrically spherical black hole and the other is a Kerr black hole, one which is rotating, and spherical. Since the Kerr black hole is much more complicated than the Schwarzschild, more emphasis will be put on describing the latter.

Formation of a Black Hole

One way a black hole forms is when a massive star collapses. Stars are basically huge continuous hydrogen bombs. The fusion reactions going on in the center of a star shoot hydrogen out towards the surface. This counteracts the gravitational pull of the star's own mass, and keeps the star stable. When the hydrogen in the center of a star runs out, the force of the gravity of the star overpowers the force of the outward fusion, because the fusion has stopped. When this happens, the star implodes, getting more and more dense. If the star is less than 1.4 solar masses, it will collapse into a white dwarf star. If it is between 1.4 and 3 solar masses, it will collapse into a neutron star. However, if the star is 3 solar masses or more, it becomes so dense and small that it reaches its critical radius (Schwarzschild radius), or event horizon, which is the point where the speed necessary for anything to reach escape velocity is higher than the speed of light, and therefore nothing

can escape, and the star becomes a black hole. As the star is collapsing, the event horizon expands outward as more mass is added to the black hole because of the definition of the Schwarzschild radius. The expansion is continuously slower, and stops when all the mass has passed through it. Before this happens, an ‘apparent’ event horizon appears:



This apparent horizon occurs because light that was trying to escape the star during the collapse, gets held back by the enormous gravity, and then no longer expands, and may even contract. This is called a trapped surface, and if it is neither expanding nor contracting, it is called the apparent horizon and cannot be outside the event horizon. Eventually the event horizon stops expanding and both horizons coincide.

Another method of how a black hole forms is believed to have only been in the early universe. This method involves extreme compression of matter by external forces, down to the critical radius of the mass of the matter. These black holes can have any mass since they aren't created by stars and are called primordial black holes. It is believed that these kinds of black holes could have formed in the early universe because conditions existed which had high enough temperatures and pressures. Primordial black holes may also account for the missing hydrogen and helium that theorists have calculated should exist after the big bang. The hydrogen and helium may have been consumed by enormous primordial black holes in the early universe.

Tidal Forces

Tidal force is the difference in gravitational force of an object. For example, from a person's head to their toe, measured in g's. The tidal forces of a black hole acting on matter increase rapidly as the distance from the black hole decreases. It follows the relation:

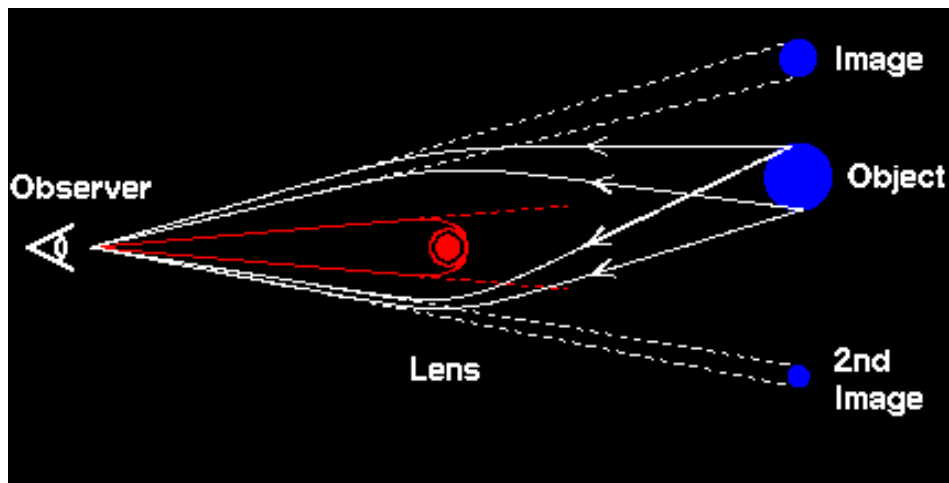
$$M/r^3$$

where r is the distance from the black hole, and M is the mass of the black hole.

The tidal forces stretch matter radially, and also compresses it laterally forming long thin strings.

Gravitational Bending of Light

All objects with any mass bend light, but it is hardly noticeable. In a black hole, the gravity is so strong that the light is bent and curved around it a lot. This creates two or more virtual images for every real object, when looked at from one side of the black hole.



The light that goes directly towards the black hole, and any light that comes to within 1.5 Schwarzschild radii of the black hole, falls into it. The light rays that are nearer to the black hole are pushed out more, so images appear compressed radially.

Orbits Around a Black Hole

A black hole is not an inescapable object. As long as anything is outside the event horizon, it is physically able to leave the pull of a black hole or orbit around it. The closest orbit that remains stable is at 3 Schwarzschild radii. Inside this distance, the orbits are not stable, and matter eventually falls towards the black hole. At 1.5 Schwarzschild radii from the center of a black hole, only light can travel fast enough to maintain an orbit. Because of this, the area at this distance only contains photons, and is therefore called the photon sphere.

The period of orbits is proportional to the mass of the black hole. However, inside the distance of 3 Schwarzschild radii, relativistic effects already start to occur making the period of orbits viewed from far away, seem longer.

For all orbits around a black hole, Kepler's 3rd law applies, but only with proper definitions of radius and period:

$$G M t^2 / (2\pi)^2 = r^3$$

where G is Newton's gravitational constant, M is the mass of the black hole, t is the period as seen from an outside observer at infinite distance, and r is the circumferential radius, so that $2\pi r$ is the circumference of a sphere at radius r . The actual period as experienced from the thing in orbit is less by a factor of $(1 - R_s / r)$, as taken from the Schwarzschild Geometry.

Schwarzschild Geometry

In 1915, Karl Schwarzschild derived a geometry to describe space-time in empty space surrounding any spherical mass:

$$ds^2 = - (1 - R_s / r) dt^2 + (1 - R_s / r)^{-1} dr^2 + r^2 d\theta^2$$

In this complex equation, $c = 1$, ds is the absolute value of the distance between two events in space and time, t is a universal time coordinate, and r is the circumferential radius as described before. This geometry is used often to describe the effects at the event horizon, and to describe the structure and curvature of space-time in the black hole.

Schwarzschild also found the evaluation of the critical radius (called event horizon) to which when any spherical mass is compressed down to the size of this radius, it would continue to collapse into a singularity at the center and become a black hole. The equation of the critical radius, or Schwarzschild radius is:

$$R_s = 2GM/c^2$$

where R_s is the radius of the event horizon, G is Newton's gravitational constant, M is the mass of the object, and c is the speed of light.

The Event Horizon

The event horizon, R_s , is the point of no return for a black hole. Once anything is inside it, it must continue on into the singularity at the center of the black hole just as time must continue to move forward. The time passed by an object falling into the horizon, as observed from far away, seems to slow down to infinity, never ever reaching the event horizon. However, this is only an optical effect caused by the rays of light. Once the object passes through the horizon, none of the light emitted can escape for outside observations. The light emitted right before crossing the threshold, becomes ever

increasingly redshifted or ‘dimmed’ until the last photon is emitted. This is because the event horizon is a lightlike surface in that light can actually stand still at the surface because general relativity shows that the surface is traveling at the speed of light; as something falls in, to its perspective, the event horizon is approaching at the speed of light right at the threshold. As the distance r increases from the horizon, more light is able to be seen because it is not redshifted or ‘held back’ as much. From the point of view of the object falling into the horizon, time passes normally and it goes through the horizon, and continues on into the singularity.

The Singularity

The singularity is the center of the black hole. Anything that falls through the event horizon ends up in the singularity. It is a point of infinite density and infinite curvature of space-time. It would seem that all common laws of physics do not exist at this singularity. This makes it very difficult to explain what happens at the singularity. New theories such as quantum gravity and superstring, may give some answers.

Wormholes and White Holes

In a Kerr black hole, it is theoretically possible for an object’s matter to avoid the central singularity. For the rotating charged black hole, timelike wormholes can act as a gateway to disconnected regions, or, other universes. At the entrance to such a wormhole, an infinite speed-up of time would occur. That is, from a particle’s point of view, it would see, in a flash, the universe accelerate in time until its end before entering the wormhole. However, as the observable universe speeds up, the light is infinitely blueshifted, which in

turn could cause a different singularity to occur from the wormhole, destroying the wormhole itself. Also, the wormholes themselves would be unstable by nature due to gravitational forces, and could fly apart. If not however, the particle would ideally travel through the wormhole and appear in a parallel universe out of a white hole, which is a black hole running backwards through time (although white holes would violate the 2nd law of thermodynamics, general relativity does not know the way cause and effect go, because it is time symmetric). Even so, this would be impossible because the particle would only be able to enter a horizon in one direction, so would become trapped in the singularity between the two holes (white and black depending on the direction of time), but it would be able to see light signals from both universes. The only way for the particle to travel between universes is if there is an exotic matter shell around the 'throat' of the wormhole threshold. This shell of exotic matter would have negative mass and positive surface pressure. The negative mass lets the throat of the wormhole lie outside the horizon, so the particle can travel through it, while the positive surface pressure prevents the wormhole from collapsing.

Hawking Radiation

Black Holes may make it seem that they will never die out, that because nothing can go faster than the speed of light, no energy or matter can escape it. However, a theory created by Stephen Hawking involving quantum mechanics may make it possible for black holes to decrease in mass and energy, and eventually evaporate completely. In the field of quantum mechanics, there is something called Heisenberg's principle of uncertainty. It states that at a specific moment, a particles position in space or it's momentum can be

measured with preciseness, but not both. This means that if greater preciseness of a particles momentum is wanted, preciseness of the position will be lost. Because of this, it can never be said that anything is zero or that a vacuum is empty. At any moment, a prediction of the creation and elimination of a particle-antiparticle pair can be made. This creation-elimination would occur in such a brief period of time, that it would be all but impossible to detect.

The theory of Hawking Radiation states that in the empty space just outside the event horizon of a black hole, these virtual particle-antiparticles can be created, and if only one of the pair of particles falls into the black hole, and the other escapes, than the particle escaping becomes real, and the particle falling in remains virtual. This virtual particle has to have a negative mass-energy due to the conservation of energy. Because of this, the black hole loses mass and energy, and eventually may evaporate completely.

Information Paradox

One important aspect of quantum mechanics is reversibility, that no matter what changes occur in particles, no matter what happens to them, that by going backwards in time, the information about them can be recovered. If this is proven not to hold true, then it implies that energy can be created or destroyed, which would violate conservation of energy.

Black holes have been called “hairless” in that the only things that determine what form a black hole has is its mass and rotation. It does not matter what it was formed from. If this is true, then information is lost in the black hole which in turn, would violate reversibility. This would cause an “information paradox”. One theory to solve the paradox

is that perhaps the Hawking Radiation which is emitted from the event horizon, carries out information from the black hole. However, this has not been proven.

Black holes will remain an object of study for countless years. A lot is now known about them, and proof of their existence has been circumstantially found. They will most likely lead to the development of new physics, especially new theories in relativity.

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