SAN DIEGO STATE UNIVERSITY

M 693 b Advanced Numerical Analysis

PDE's in Curvilinear Coordinate Systems

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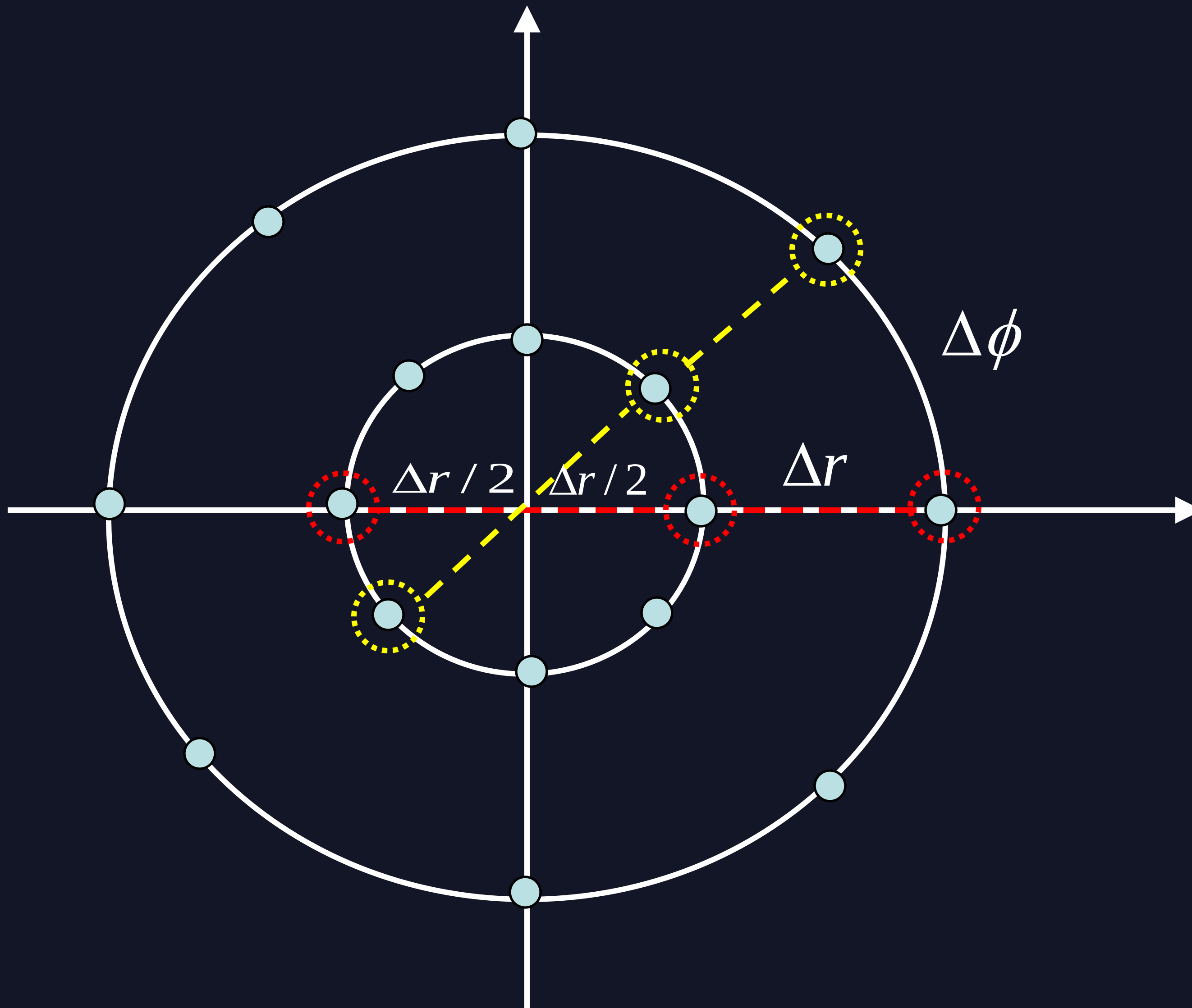
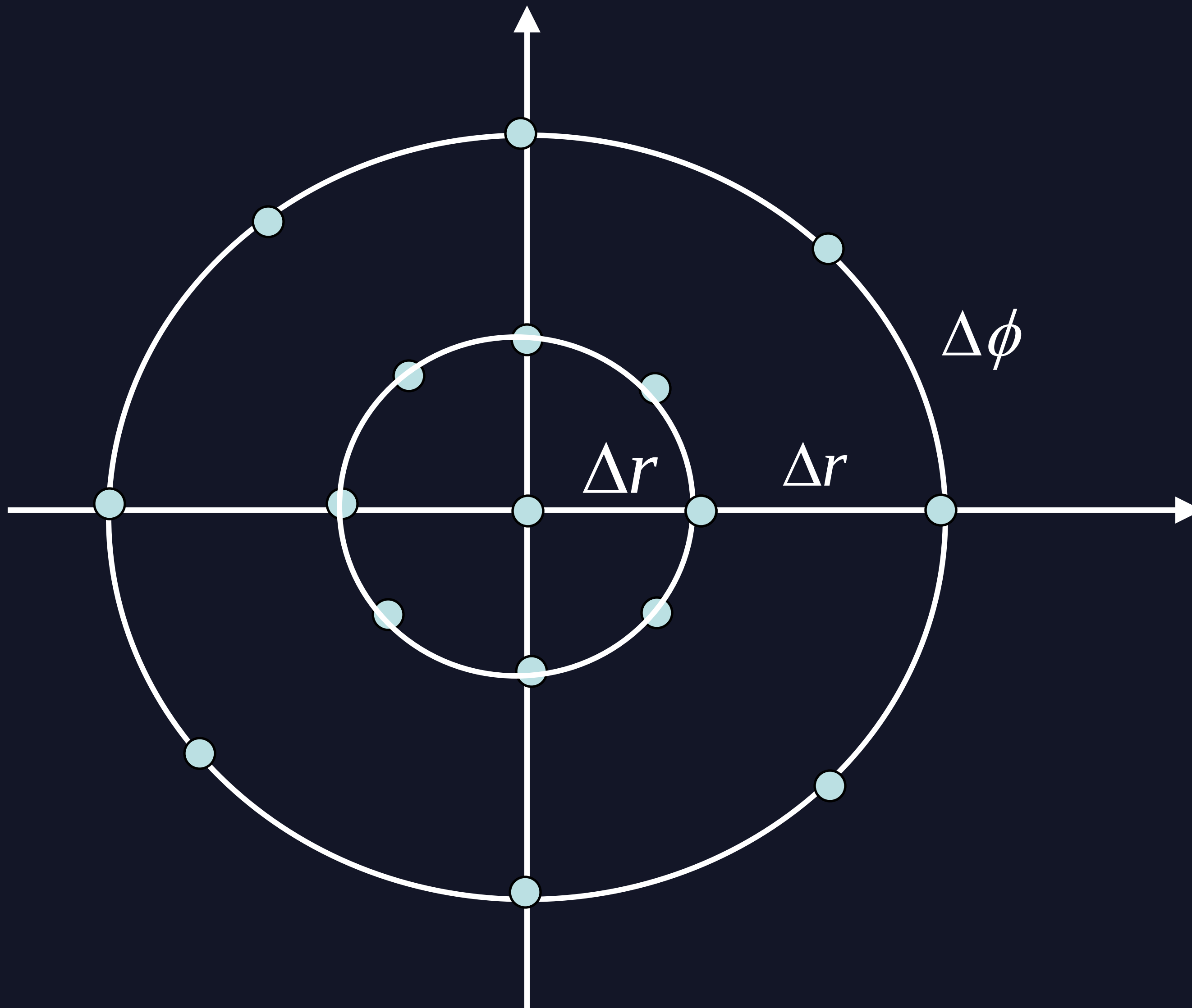
Overview

- Grids in curvilinear coordinates
- Poisson Eq. in Polar Coordinates
- Wave Eq. in Polar Coordinates
- Heat Eq. in Polar Coordinates
- Heat Eq. in Spherical Coordinates

Grids in curvilinear coordinates



Grids in curvilinear coordinates
Another Approach



Poisson Equation

- Poisson Equation

$$\nabla^2 u(x,y) = f(x,y)$$

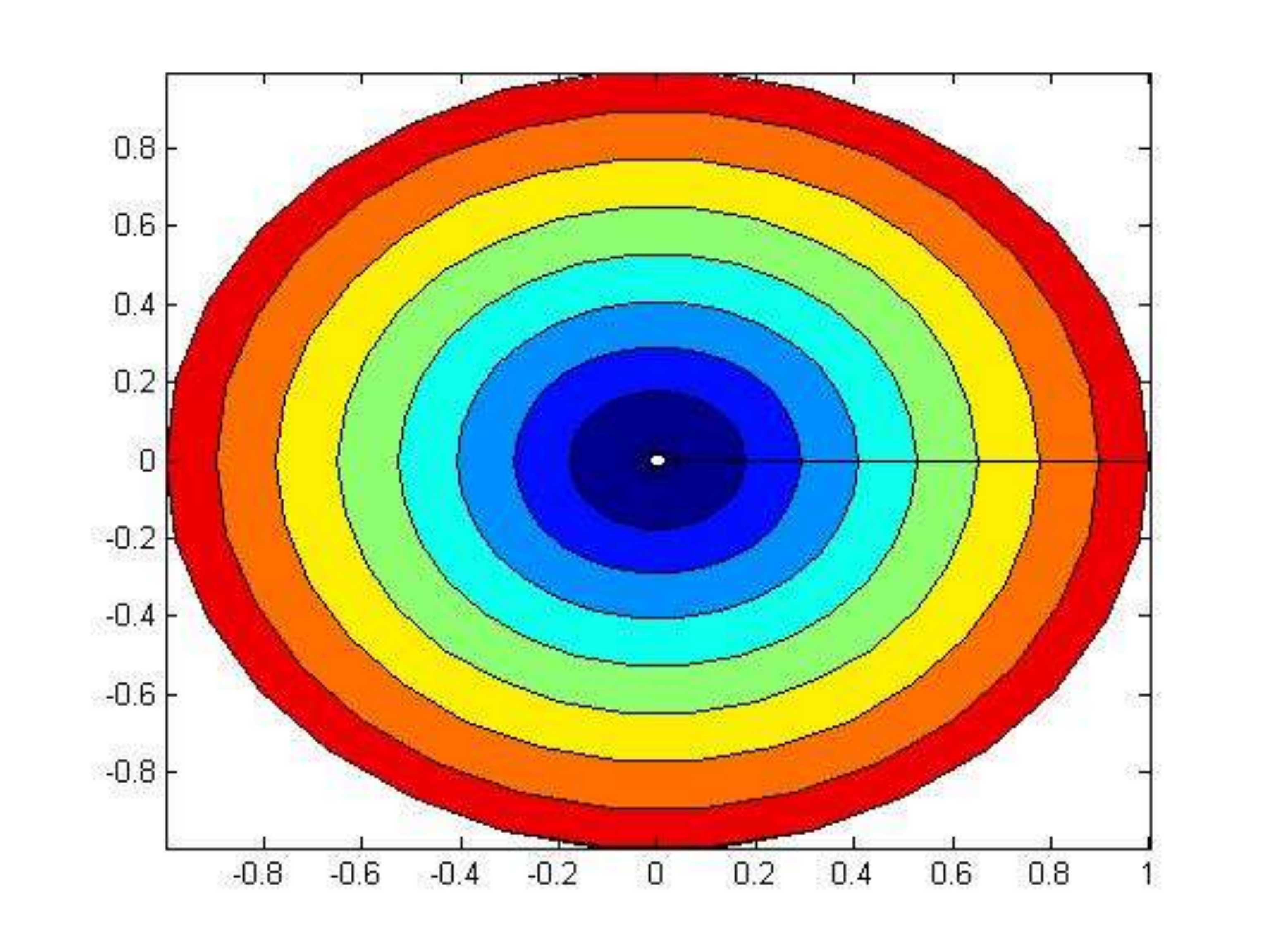
- Poisson Equation in Polar Coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{1}{r^2}\left(\frac{\partial^2 u}{\partial \theta^2}\right)= f(r,\theta)$$

Poisson Equation

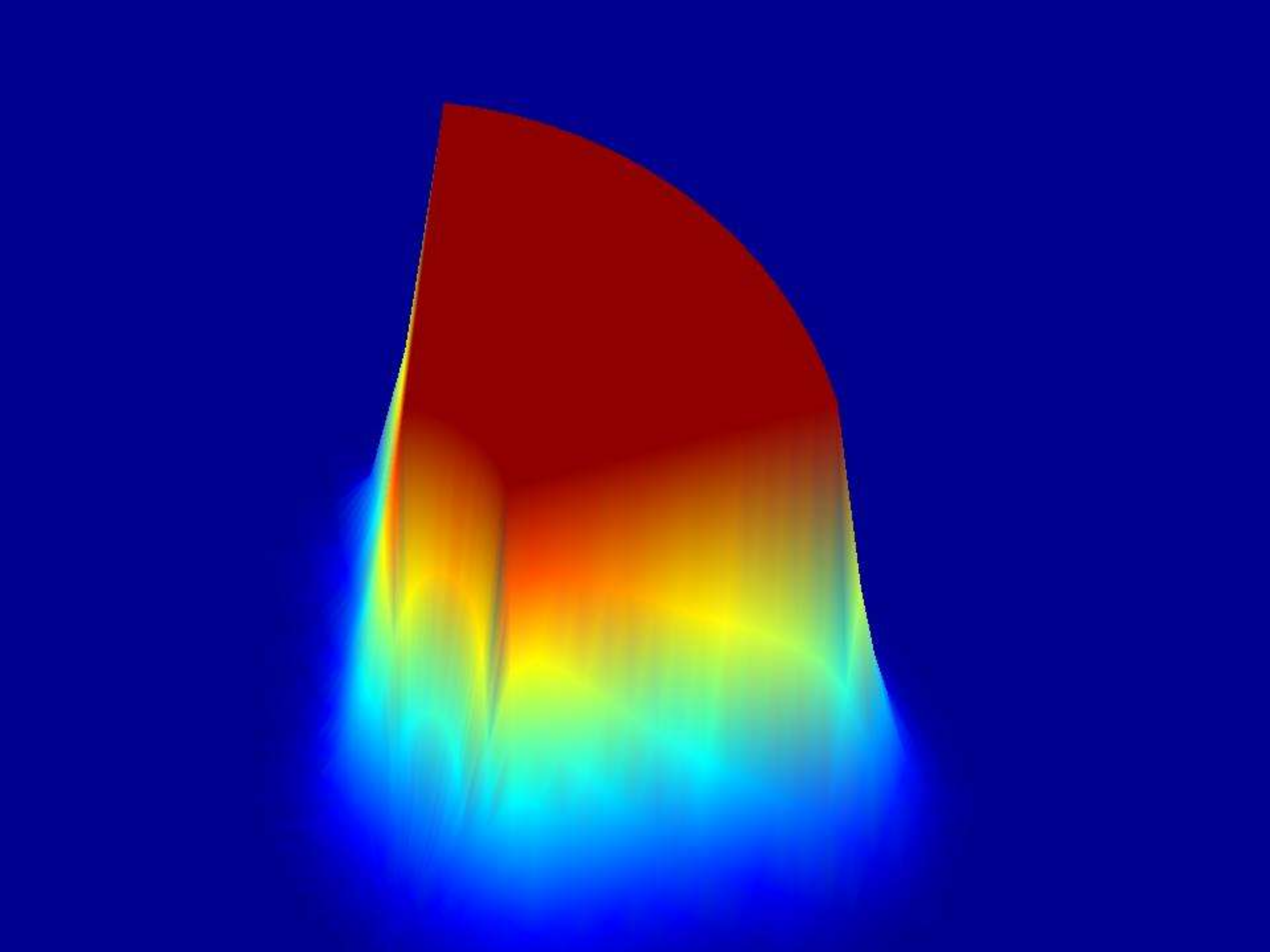
- Five point Laplacian
- Conjugate Gradient Method
- Round off errors ???

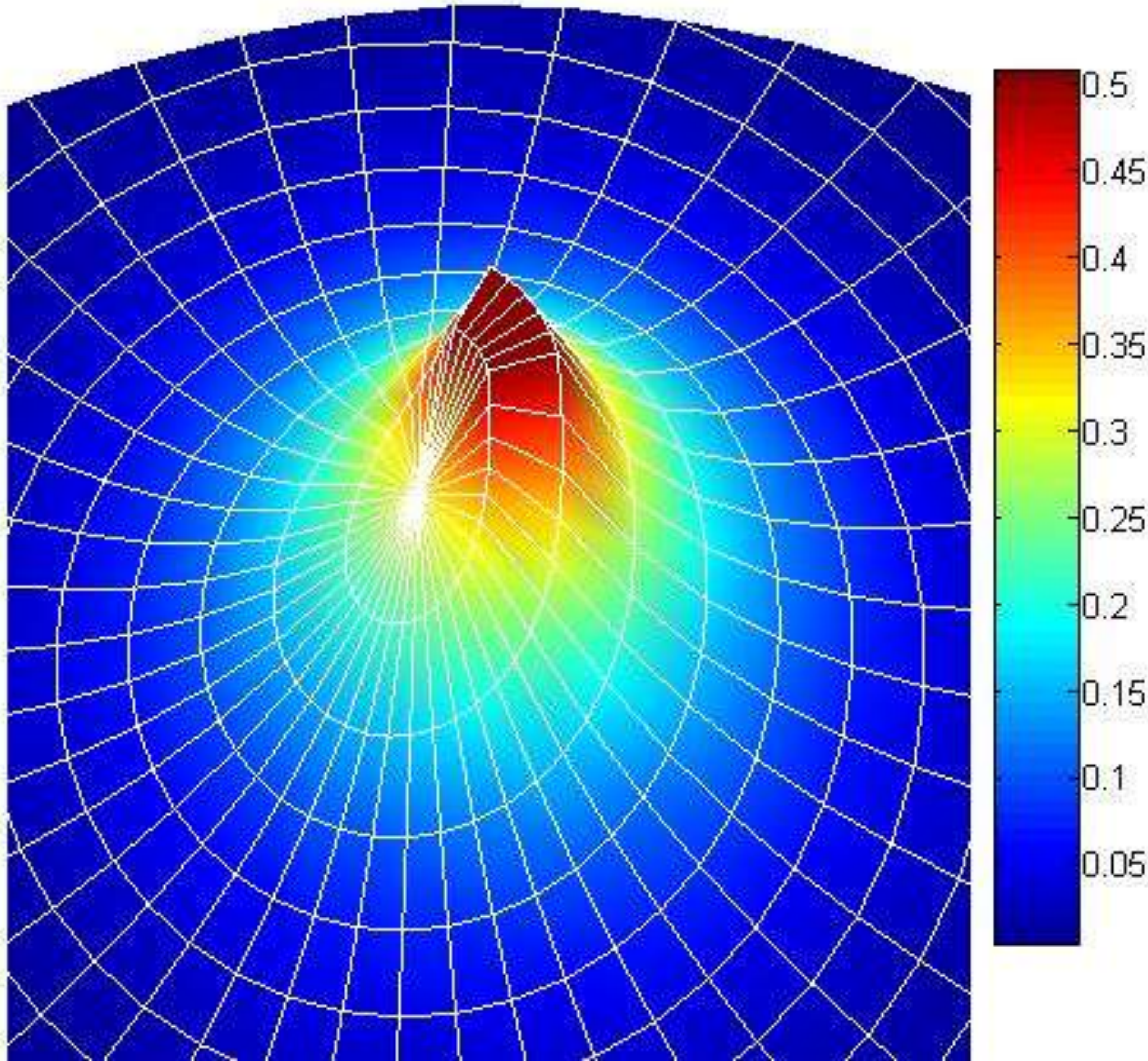
Results



2D Heat Equation on a Disk
Ut = b(Uxx + Uyy)

- New R=0 Approach
- Different Stability Requirements R vs. Phi
- Boundary Conditions
- Not Symmetric

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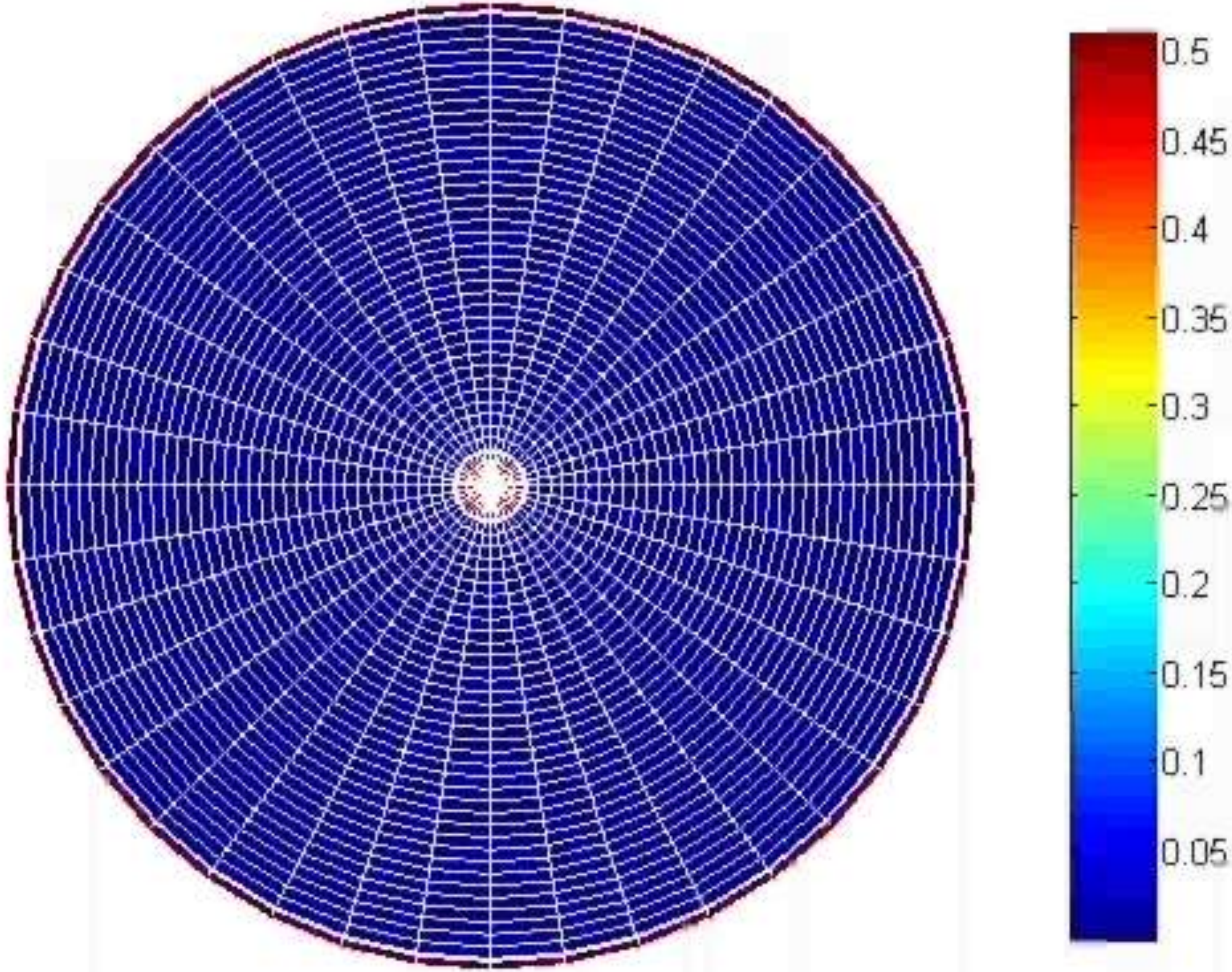
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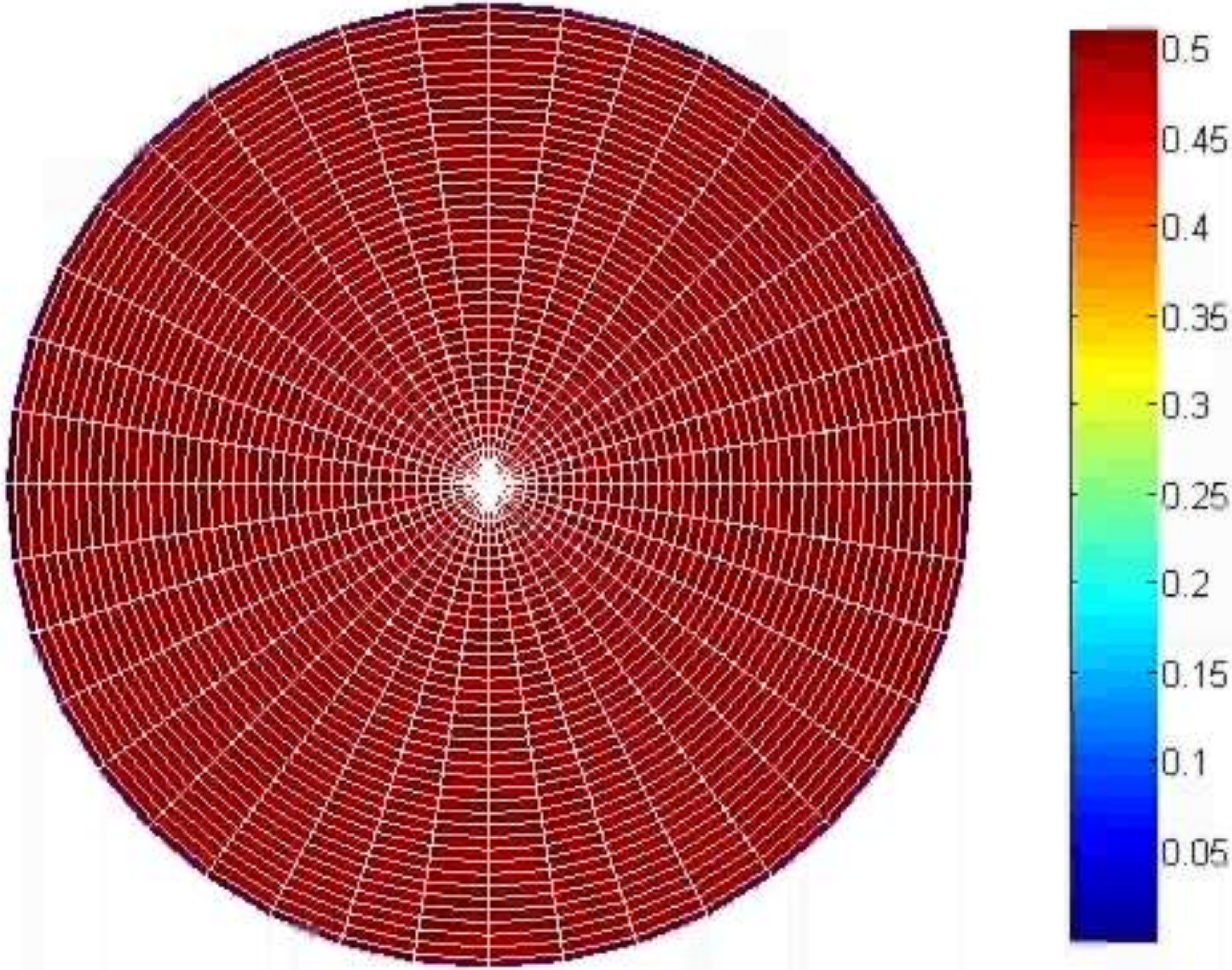
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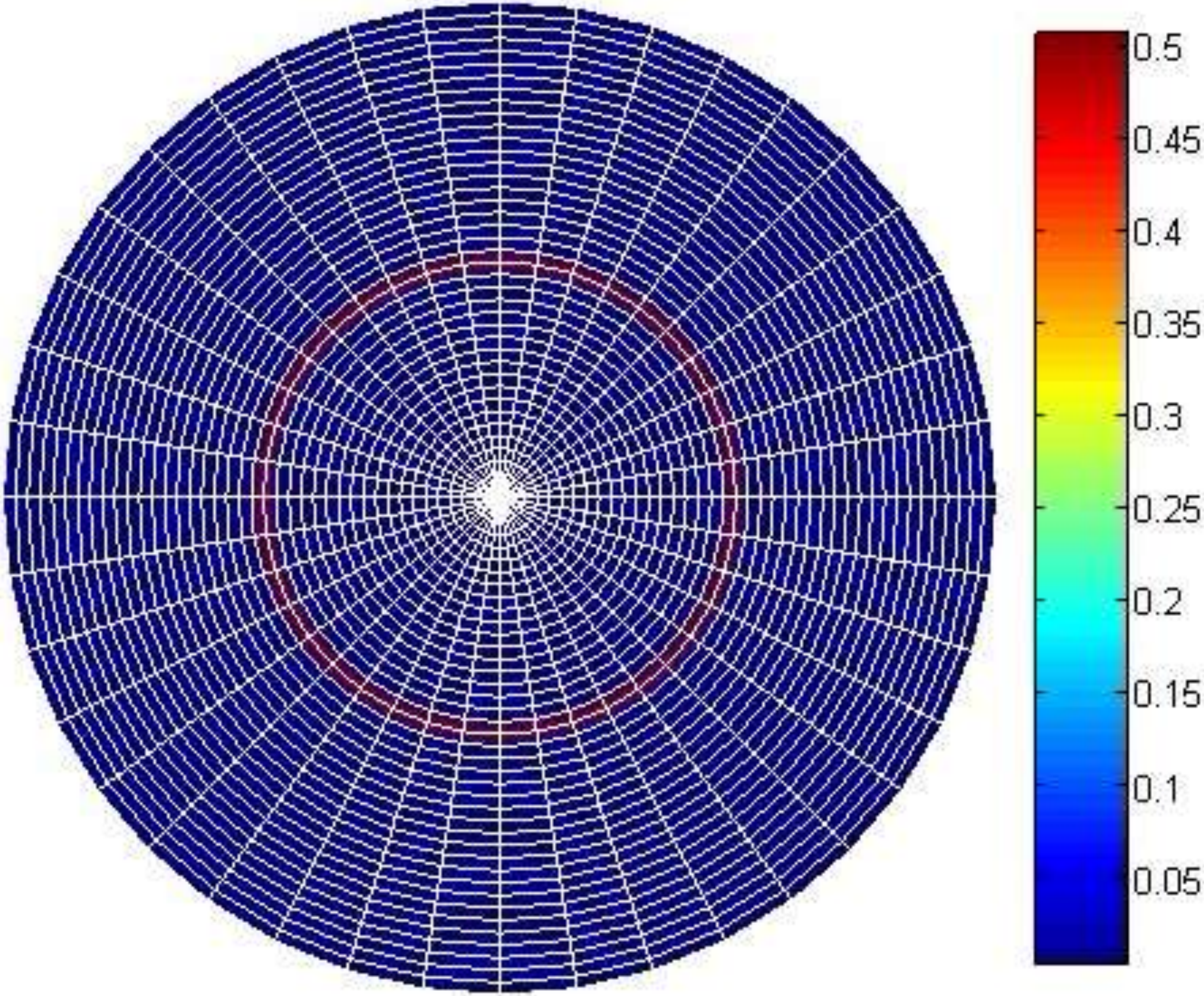
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Wave equation in a disk

- The second order wave equation is given by

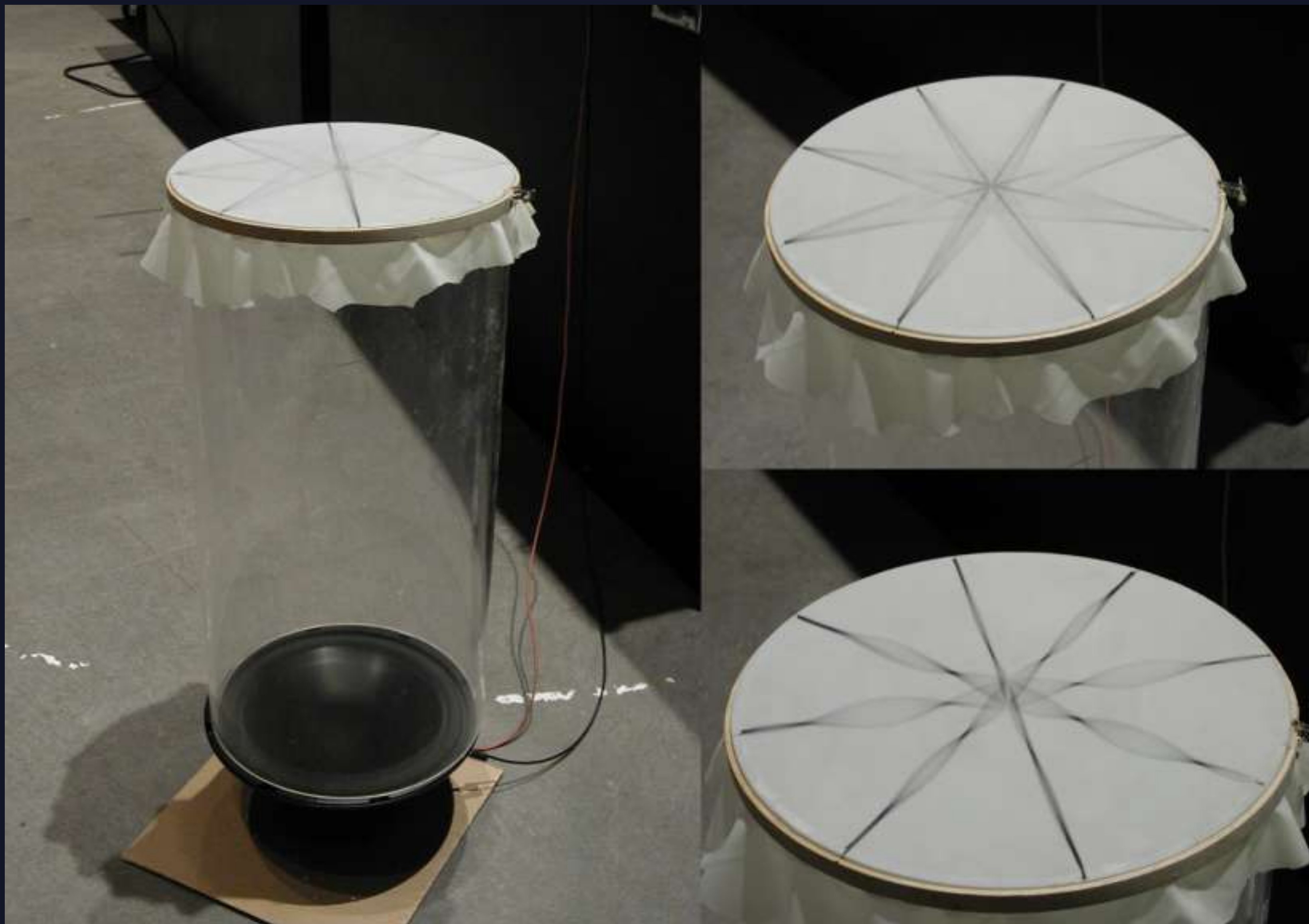
$$\frac{\partial^2 \psi}{\partial t^2} - \frac{1}{c^2} \nabla^2 \psi = 0$$

- In polar coordinates last eq. becomes

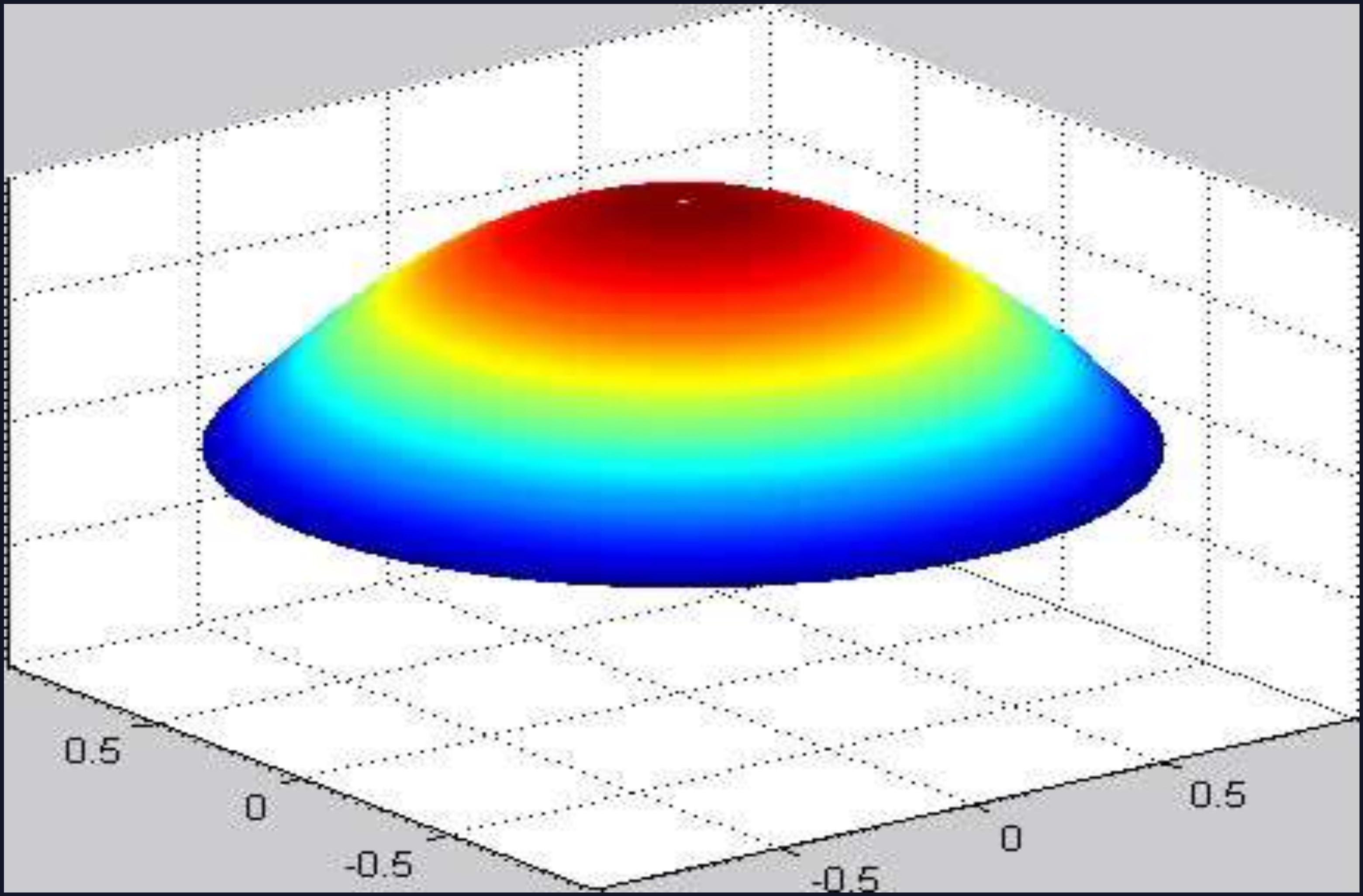
$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \phi^2} \qquad (c = 1)$$

Drumhead

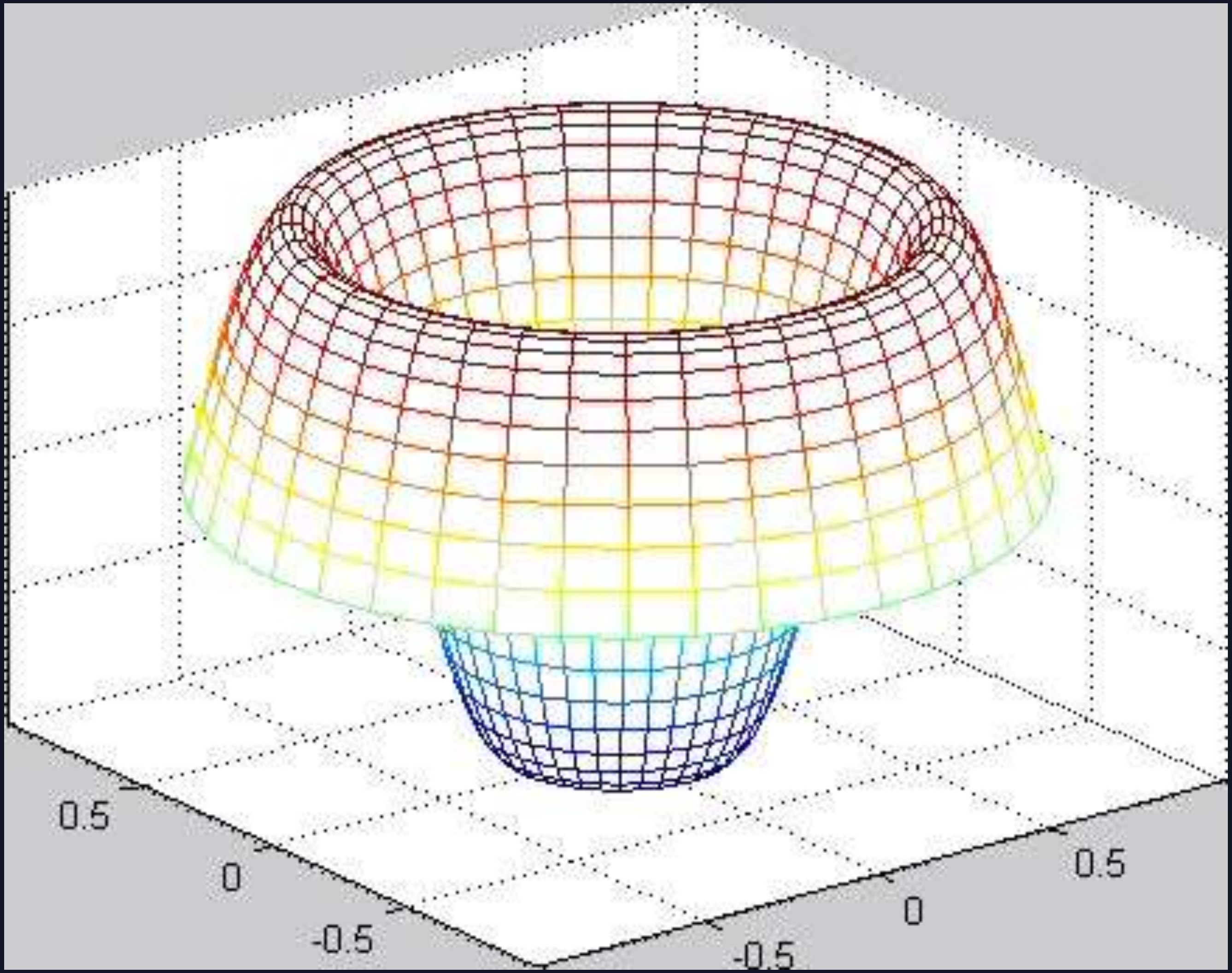
$$\psi(R) = 0$$



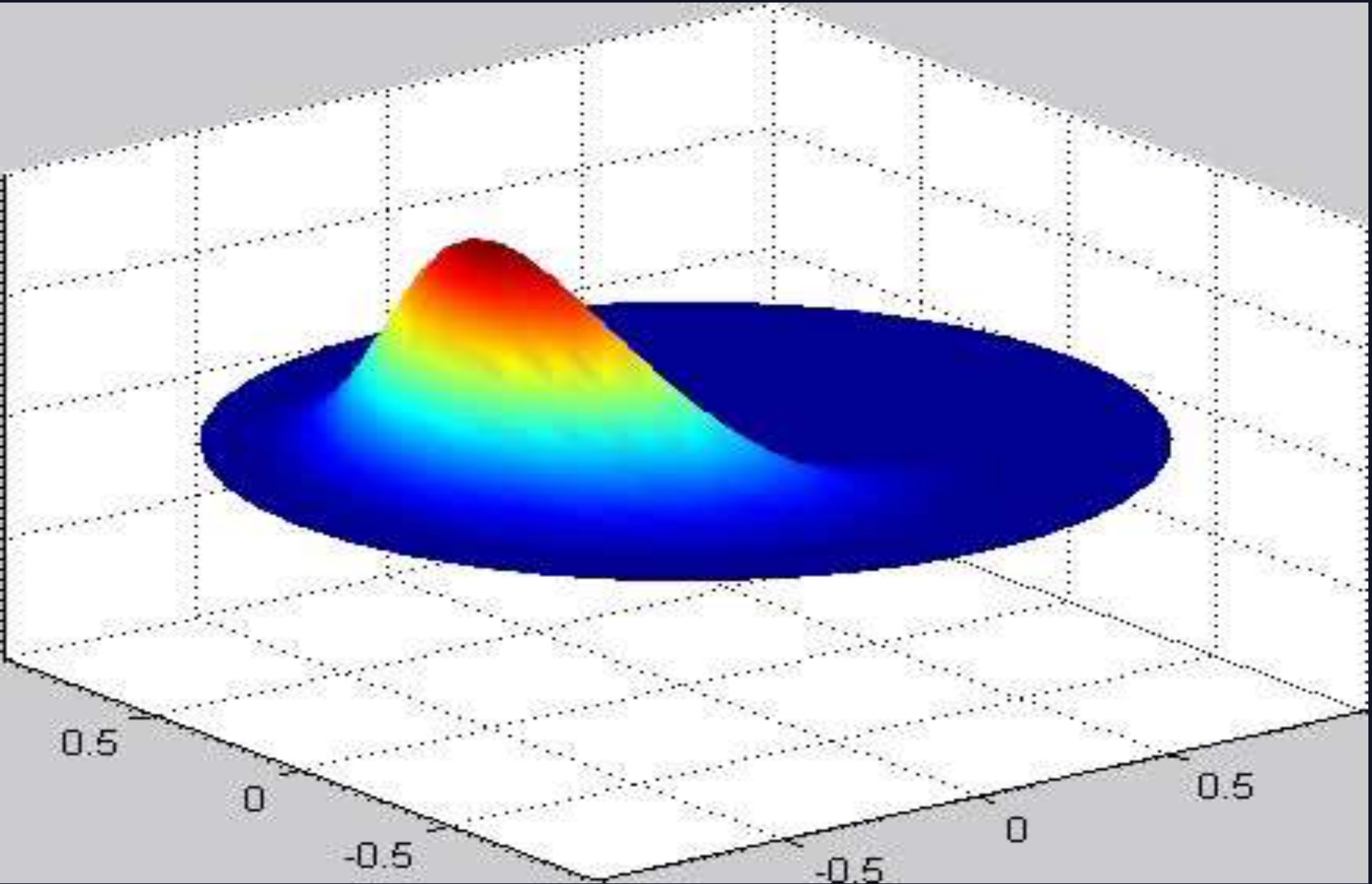
Drumhead - results



Drumhead - results



Drumhead - results



Drumhead - remarks

- Method very stable for smooth initial conditions.
- Extremely sensitive to non-smooth initial conditions: requires very fine grid.

Heat equation on the surface of a sphere

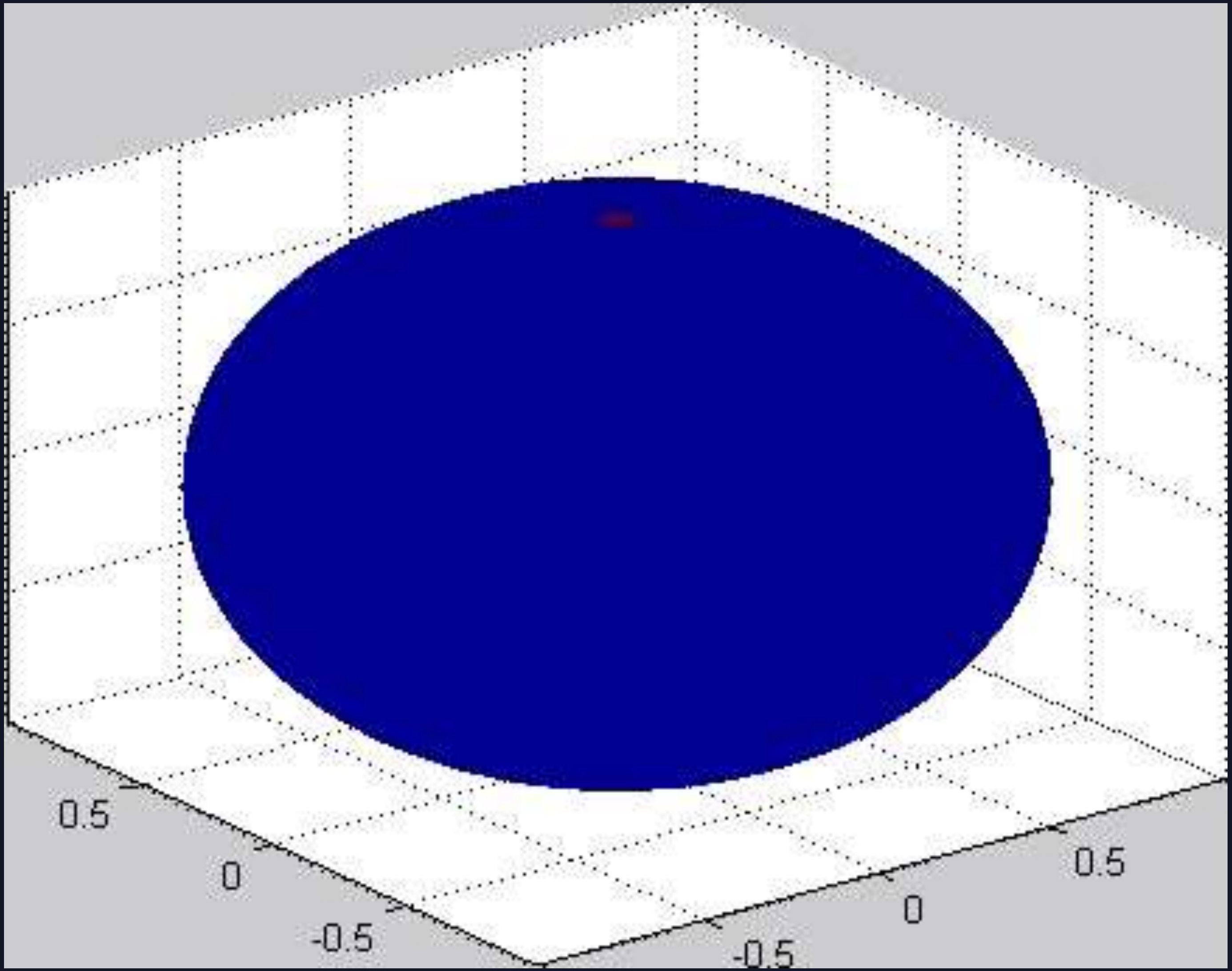
- Heat equation

$$\frac{\partial T}{\partial t} = k\nabla^2 T$$

- On the surface of the sphere

$$\frac{\partial T}{\partial t} = \frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{1}{R^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{\cot\theta}{R}\frac{\partial T}{\partial \theta}$$

Results



Results